

# **Economic Capital**

Implementing an Internal Model for  
Economic Capital



NUMERICA

ACTUARIAL SERVICES

# ABOUT THIS DOCUMENT

## THIS IS A WHITE PAPER

This document belongs to the 'white paper' series authored by Numerica. It provides concise and general direction on specified issues.

## OBJECTIVE OF THIS PAPER

This paper focusses on implementation of an EC framework in a life insurance company. The first part discusses the introductory concepts and the second part sets out a roadmap for implementation of the EC model.

EC is an extensive subject and this article doesn't go into full technical details on all the issues. Instead, this paper focusses on implementation; especially on making the high-level decisions. This article assumes some basic understanding of EC and statistical distributions.



## INTRODUCTORY CONCEPTS

- What EC implementation entails?
- Where to start developing an EC framework?
- Building an EC model based on Solvency II
- Calculating EC as per Pillar 1 principles
- Choosing between SF and IM approaches



## ROADMAP TO BUILDING AN EC INTERNAL MODEL

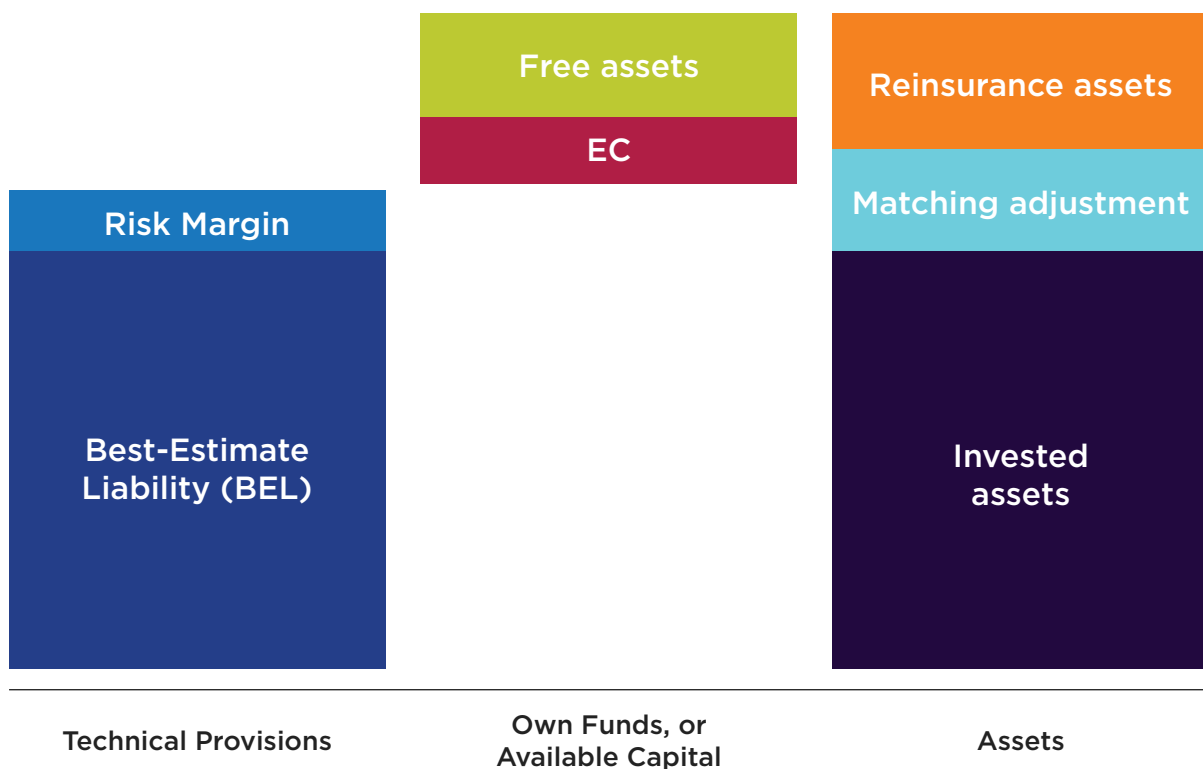
- Risk identification
- Risk calibration
  - Defining risk measures
  - Calibrating stresses or risk distributions
  - Estimating correlations and calibrating a copula, if needed
- Aggregation
  - Correlation matrix approach
  - Copula approach
- Monte Carlo simulation
  - Proxy modelling
  - Concluding remarks

# 1 INTRODUCTORY CONCEPTS

## What EC implementation entails?

An EC framework is not limited to implementation of capital requirements alone. Rather, the entire balance sheet of an insurer is drawn up, and several new elements are introduced.

Let's take a brief look at what an EC balance sheet looks like:



This figure would seem quite familiar to anyone who has some knowledge of EC, Risk based capital (RBC) or Solvency II, and the concepts are well understood.

Instead of going on to explain each of these blocks, it is best to highlight just a few points at this stage:

- Under an EC framework, reinsurance assets are disclosed separately on the balance sheet. Under many regulatory regimes, reinsurance assets are completely omitted and the liabilities are disclosed net of reinsurance.
- Reinsurance assets are calculated after allowing for 'expected defaults' by the reinsurers.
- Matching adjustment arises when long-term business (e.g. annuities) is backed by assets that earn credit spread, over and above the risk-free rate. Examples of such assets include corporate bonds, mortgages, equity release. Rather than shown separately as an asset, matching adjustment is most commonly included within the Technical Provisions as a negative liability.

## Where to start developing an EC framework?

The first step is to define your EC. If the EC is part of a wider ERM framework, the definition of EC will be in line with the 'risk appetite' statement set out by the Board of Directors. If such a statement has not been set out by the Board, the EC can be defined in isolation. However, an EC which is not part of an ERM framework is likely to be of limited use.

The definition most widely used is:

*"To remain solvent with a probability of 99.5% over a 1-year period."*

In this case, the EC is defined as the amount of capital the company will need to keep to stay solvent over the next one year with a probability of 99.5%. This is also an example of 1-year VaR (Value-at-Risk) approach.

Companies can go a step further and define EC as:

*"To remain solvent if a 99.5th percentile stress event was to occur over a 1-year period."*

This definition aims to keep enough capital to remain solvent even if a 99.5th percentile stress event was to occur. The EC in this case will be based on Tail VaR (TVaR), also called Conditional Tail Expectation (CTE).



## Building an EC model based on Solvency II

The concept of EC is not new. Insurers and their regulators around the world have been working towards making sure that their businesses are run on a robust EC framework. Solvency II could be a potential starting point. Solvency II is a European solvency framework for all insurers based on EC and ERM principles and has been in place successfully since 1 Jan 2016. Solvency II is a 3-pillar framework and the first pillar deals with estimating Solvency Capital Requirements (SCR; equivalent to EC).

## Calculating EC as per Pillar 1 principles

An insurer can choose to adopt Solvency II Pillar 1 principles for developing an EC framework. The Level I and Level II (Delegated Acts) texts set out the principles on which the SCR needs to be calculated. These principles need to be followed by companies developing their own internal models (IM) for EC calculation. However, Solvency II also provides an option to adopt a Standard Formula (SF) approach. Under the SF approach, SCR is calculated as the loss under several different pre-defined stresses to risk factors and then aggregating the losses using a correlation matrix. This differs from the IM approach, in which the insurer chooses the approach, the level of stresses, parameters etc.

## Choosing between SF and IM approaches

SF is primarily designed for small European insurers, since the cost of IM development is often substantial. Leaving the cost factor aside, few issues need to be considered for SF:

- SF has calculation modules for most risks, but not all. For example, there is no separate module for inflation risk, which is allowed for implicitly in real interest rates.
- SF prescribes stresses for each risk measure. These stresses have been calibrated using an extensive Europe-wide exercise and have been kept at a level to ensure that capital requirements are not understated. Therefore, SF may not provide a true EC for any insurer in Europe, and more so for insurers doing business outside Europe.
- Even for companies basing their EC on Solvency II SF, it may be necessary to estimate stresses independently to better reflect the economics of their own markets.
- Alternatively, insurers may develop their own IM from ground up, making sure that the principles of Solvency II are all met.

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## ROADMAP TO BUILDING AN EC INTERNAL MODEL

This section sets out the process of developing an Internal Model based on Solvency II Internal Model principles. The process would involve two or four broad steps, as set out in the paragraphs below.

### 2.1 Risk identification

The first step would be to identify risks facing the insurer. Usually, this would involve one or more of the following:

- Analysis of the balance sheet to identify the exposures on both asset and liability sides. E.g. presence of a currency swap on the balance sheet could point to the existence of currency risk.
- Analysis of income statement to uncover any losses arising from off-balance sheet exposures. E.g. a litigation expense could point to reputation risk.
- Risk identification checklist of the UK actuarial profession.
- Interviews, brainstorming and workshops

Generally, the following risks will exist for a typical life insurer:

- Interest rate risk
- Credit spread risk
- Inflation risk
- Equity risk
- Property risk
- Currency risk
- Counterparty default risk
- Mortality, or longevity risk (level, trend, volatility)
- Persistency risk
- Expense risk
- Operational risk





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## 2.2 Risk calibration

Having identified the risks, the next step is to calibrate those risks.

### 1. Defining risk measures

The calibration process requires a 'risk measure' to be defined for each risk. To start with, a risk measure could be an assumption in an asset or a liability model; e.g. persistency, mortality, valuation interest rate. But sometimes the risks could be more complex. For example, interest rate risk is best represented by Principal Components (PC), rather than a single rate for EC purposes.

The risk measure is the metric for which historical values will be extracted and statistical analysis will be performed. The risk measure for equity risk could be set to NIFTY total return. The historical values of NIFTY total return can then be used for statistical analysis.

The table below provides examples of risk measures for some of the key risks:

Risk	Risk measure examples
Interest rate risk	<ul style="list-style-type: none"><li>• Principal component multipliers (complex)</li><li>• Spot rate at the average settlement duration (simple)</li></ul>
Credit spread risk	<ul style="list-style-type: none"><li>• Average credit spread on own portfolio (complex)</li><li>• iBoxx index of corporate bonds (simple)</li></ul>
Equity risk	<ul style="list-style-type: none"><li>• Total return on own equity portfolio (complex)</li><li>• NIFTY Total Return index (simple)</li></ul>
Mortality risk	<ul style="list-style-type: none"><li>• Historical claim counts expressed as a percentage of a mortality table</li></ul>
Persistency risk	<ul style="list-style-type: none"><li>• Historical surrenders split by categories (complex)</li><li>• Overall historical surrenders expressed as a percentage of the pricing assumptions (simple)</li></ul>

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## 2. Calibrating stresses or risk distributions

Once the risk measures have been identified, the next step of the calibration process involves one of the following, depending on the choice of IM architecture (see next point):

- Estimating the 99.5th percentile stress for the risk measure (usually referred to as the ‘biting scenario’)
- Estimating the entire loss distribution for the risk measure

**It is important to note that the real-world probabilities distributions or stresses are produced during this process.** Risk-neutral probabilities are not directly used in EC calculation.

Risk calibration is an extensive statistical process with its own set of challenges, issues and considerations, which are not discussed in this paper. These could include choice and selection of data, choice of distributions for testing, selection criteria and parameterisation approach.

## 3. Estimating correlations and calibrating a copula, if needed

Various measures of correlation exist and which one to use depends on the aggregation method (see next section). Generally, one of the following two types of correlations need to be calculated:

- **Pearson’s correlations:** this is the most familiar type of correlation which measures linear correlation between two random variables.
- **Spearman’s rank correlations:** each random observation is assigned a rank and then the correlation between the ranks is calculated using the usual formula.

Risk		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Interest rate PC1	(1)	1.0	0.0	0.0	0.2	-0.2	0.0	-0.3	0.0	0.0
Interest rate PC2	(2)	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Interest rate PC3	(3)	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
Credit spread	(4)	0.2	0.0	0.0	1.0	-0.1	0.0	0.0	0.0	0.0
Equity	(5)	-0.2	0.0	0.0	-0.1	1.0	0.0	0.0	0.0	0.0
Mortality	(6)	0.0	0.0	0.0	0.0	0.0	1.0	-0.3	0.0	0.0
Persistency	(7)	-0.3	0.0	0.0	0.0	0.0	-0.3	1.0	0.0	0.0
Expense	(8)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
Counterparty	(9)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0

The correlation matrix needs to be converted to Positive Semi-Definite (PSD), so the correlations are mutually consistent.

If multivariate approach is used, calibration of a copula will also be required (see next section).



## 2.3 Aggregation

Having calibrated the risks, the next step is to decide how to use those risk distributions to calculate the EC.

This will perhaps be the most important decision to be made for developing an EC model. Broadly, an IM is generally of one of the following two types: the one that uses a correlation matrix, or using a multi-variate technique.

### (A) Correlation matrix approach

Under this approach, the balance sheet of the insurer is re-calculated by changing one assumption at a time, from their base value to the calibrated 99.5th percentile value. The loss, which is the decrease in surplus (i.e. decrease in excess of assets over Technical Provisions), is calculated.

These univariate losses are calculated for each risk measure. The losses are then aggregated using a correlation matrix. A simplified example is shown below:

Risk	99.5%ile loss
Interest rate PC1	35,000
Interest rate PC2	15,000
Interest rate PC3	5,000
Credit spread	20,000
Equity	45,000
Mortality	15,000
Persistency	10,000
Expense	2,000
Counterparty	1,000
<b>Sum of standalone risks</b>	<b>148,000</b>
Diversification benefit	(89,690)
<b>Economic capital</b>	<b>58,310</b>



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The aggregation is based on the correlation matrix in the previous section.

The correlation matrix approach has the primary advantage that it is easy to follow and implement. However, it suffers from several drawbacks:

- This method assumes that the loss functions follow normal (to be precise, elliptical) distributions. However, most risks do not follow a normal distribution and the corresponding loss functions will be non-normal as well. Therefore, separate adjustments will generally be required to allow for non-normality and the magnitude of this adjustment can be quite significant.
- Normal distribution is a thin-tailed distribution, which means it tends to under-estimate losses that are of low frequency, but high severity. Therefore, the correlation matrix approach is likely to under-state the EC.
- The 99.5th percentile point of the loss distribution is all that matters. Rest of the distribution is not usually used.
- It makes no allowance for increased correlations between risks in times of stress (also referred to as 'tail' dependence).
- It is assumed that there are no interactions between risks.
- It is assumed that losses vary linearly with the risk measures.

Also, note that:

- The correlations used in this method should be the Pearson's correlations.
- If the correlation matrix approach is used, the process of estimating capital requirements essentially ends here, except if making adjustments such as non-linearity and non-normality.



<sup>1</sup>A distinction must be drawn between a risk distribution and a loss function for that risk. While the former is the probability distribution of the risk measure chosen to represent that risk, the latter is the distribution of losses when the balance sheet is subjected to changes in the corresponding risk measure. Loss functions, rather than risk distributions, are used for aggregation.

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## (B) Copula approach

A more advanced alternative to using the correlation matrix approach is to use a copula for aggregating the risks.

A copula can be thought of as a 'multivariate probability distribution' of all risk measures together. In other words, it is the combined distribution of all marginal (i.e. individual) risk distributions. Since the marginal distributions can be combined in several ways, there could be multiple copulas for the same set of marginal distributions; e.g. Gaussian copula, t-copula etc.

Despite the increased complexity, the copula approach offers the following advantages over correlation matrix approach are:

- There is no need to assume that the risk distributions are normal. The risks can follow any distribution.
- With the right choice of a copula, the method can deal with the increased correlations between risks in times of stress.

Aggregation using a copula works as follows:

- Calculate **rank** correlations between the chosen risk measures by analysing the past data. These correlations between risk measures are referred to as the 'input' correlations.
- Choose a copula for aggregating the marginal risk distributions. For example, a t-copula, which allows for interaction between risks in times of stress.
- For using t-copula, a parameter called degrees of freedom needs to be calibrated. This can be done by carrying out a multivariate regression on all the risk measures together.
- The next step is to produce a large number of correlated scenarios using the calibrated copula. This can be done by generating random numbers from a uniform distribution for each risk measure, and applying Cholesky decomposition to the correlation matrix to produce correlated random numbers between 0 and 1. The correlated uniform random numbers are then inversed back into values of corresponding risk measures.

An example of the output produced from a copula simulation is as follows:

#	Interest rate PC1	Interest rate PC2	.....	Equity	Mortality	Persistency
1	0.5	0.35		+5%	+3%	-5%
2	0.2	-0.10		-7%	-1%	+1%
...						
n	-0.1	0.10		-12%	+5%	-3%

Table: Example of the scenarios produced using a copula

<sup>3</sup>The 'output' correlations, by contrast, are those that are exhibited by the loss distributions.

## 2.4 Monte Carlo simulation

Monte Carlo simulation will not be required if the correlation matrix of aggregation has been used. It is used to construct the entire loss distribution of the insurer under a copula approach.

The output of the copula process is a set of scenarios (iteration); each scenario represents a set of values of 'all' risk measures, like drawing observations from a multivariate distribution. The balance sheet of the insurer is recalculated under each scenario.

Scenario #	Loss
1	5,000
2	-2,000
...	
n	7,000

Table: Calculated losses under each scenario

The output of the Monte Carlo simulation is the overall loss distribution (referred to as Probability Distribution Forecast under Solvency II). EC is calculated as the loss at 99.5th percentile as shown in the chart below:

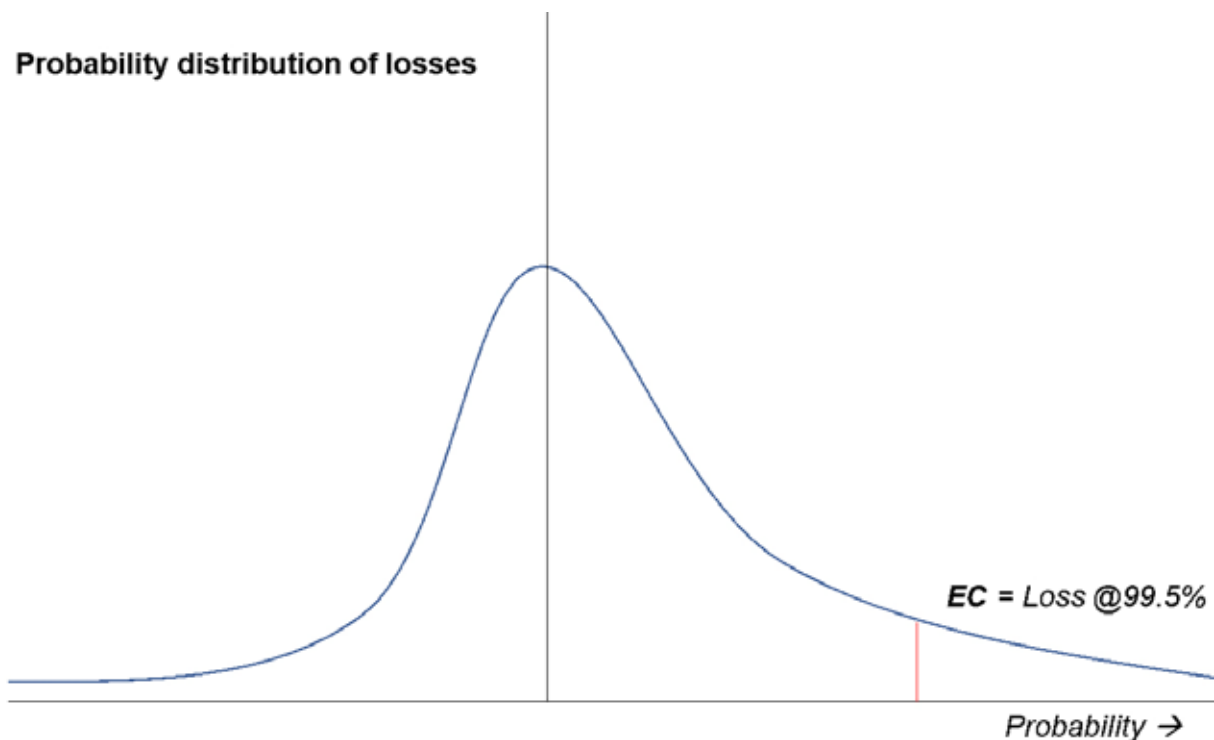


Figure 1: Probability Distribution Forecast (Loss distribution)

## Proxy modelling

Monte Carlo simulation involves recalculating the value of liabilities and assets under potentially several hundred thousand scenarios. While asset values can be readily calculated under each of these scenarios, performing liability valuation under each scenario is likely to be impractical, given the computing power currently available. This is where the concept of proxy modelling comes into picture.

The idea behind proxy modelling, is to replace the liability models by polynomial functions. These polynomial functions are referred to as 'proxy models' or 'light models'. For distinction, the liability models themselves are referred to as 'heavy models'.

Constructing the proxy models involves the following steps:

- Decide on the level of granularity of proxy models – how many proxy models need to be developed. Proxy functions are usually developed for each product line, and/or any other category deemed to be homogenous in terms of risk characteristics.
- Decide which risk measures will feature in each proxy model. Each proxy model may contain tens of terms. Each term either represents the impact of a risk measure (e.g.  $Ax^3$ , A is the coefficient of the term and x is a risk measure), or the impact of an interaction between risks (e.g.  $Cx^2y$ ). An example of a proxy model having just two risk measures (x and y) is shown below:  
 $Liability = Ax^3 + By^2 + Cx^2y + Dxy^2 + K$
- Specify calibration and validation scenarios for which the exact liability valuation will be performed using the heavy models. Calibration scenarios are used for calibrating the proxy functions and validation scenarios are used.
- Calibrate the proxy functions (i.e. estimate the coefficients and order of the polynomials) and assess the accuracy of fit.
- If the quality of fit is found to be inadequate, repeat the process by changing the order of the polynomial, recalibrate and re-validate.

## Concluding remarks

Building an Internal Model for EC is a long journey. The first step is to have a clear line of sight of the process and having a vision of the outcome, which is the main focus of this paper. Many important decisions need to be made along the way. All these issues and challenges cannot be discussed in one paper, but with the high-level overview provided in this paper, the readers can get a sense of what the journey is going to look like.

Calculation of other aspects of the EC balance sheet are due for the next article.





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